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A DYNAMIC ANALYSIS OF ALOHA SYSTEMS WITH BLOCKING AND CARRIER SENSE

ABSTRACT

A mathematical model of a proposed scheme to implement an ALOHA system with blocking and carrier sense is analyzed. New packets and blocked packets are each attempted to be sent at times which form a Poisson point process. A system of first order differential equations which describe the dynamic behavior of the sytem is obtained. These equations can be solved by conventional numerical techniques. If a steady state exists, the differential equations become a set of linear equations and a steady state solution is found with additional rate and normalization equations. Steady state solutions of simple examples for one and two user systems are given. It was found that throughput may approach $\frac{1}{\tau}$, but in that operating condition, the average delay approaches $N\tau$ and the users are probably blocked most of the time. By reducing throughput, however, the average delay and average number of blocked users can be reduced.

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TABLE OF CONTENTS

Abstract i

Table of Contents ii

I. Introduction

A. Background 1

B. Model Description 3

II. Analysis

A. Carrier Sensing with Blocking for a Single User System 6

B. Carrier Sensing with Blocking for an N User System 14

C. Solution of the Differential Equation 23

D. Steady State Solution 27

E. Efficiency Parameters 32

III. Results and Discussion

A. One and Two User ALOHA System 34

B. N User ALOHA System 39

C. Conclusions 40

Acknowledgement 41

References 42

I. INTRODUCTION

A. Background

The ALOHA system is a radio communication system for multiple computer input/output terminals called users which are linked to a central processing computer through the same radio channel. Information is sent in fixed length digital strings called packets which include the sender's identification and parity checks.

Since any one user does not send packets continuously, the channel can be shared by several users. The main problem with a time shared channel is the condition where more than one user sends at the same time called a collision. The information in the colliding packets becomes scrambled together and undecipherable. Thus, in any operating scheme in which collisions may occur a retransmission policy is necessary.

Several operating schemes have been proposed to implement such time sharing channels. Among the various schemes are: 1) an acknowledgement-only scheme where all users send at random times but must receive a confirmation that their packet was the only one being sent before proceeding to send the next packet, 2) a slotted time-acknowledgement scheme where all users send only within a random time slot and also require a confirmation, and 3) a time division multiplexed (assigned slot) scheme where each user can send only during his assigned time slot. The first two schemes require more computer time to send the confirmations and the users need a retransmission policy since collisions may occur. Collisions are impossible in the third scheme but all users must be synchronized to a central clock.

This paper analyzes another proposed scheme which uses carrier sensing and blocking. In this scheme every user tries to send at random times but is allowed to send a packet only if the channel is not being used. This requirement prevents packet collisions and does not require a central clock for synchronization. However, when a blocked packet condition occurs because the channel is busy when a user tries to send, the user must try to send repeatedly until he successfully sends his packet when the channel is not busy. In all of these schemes, the computer replies and outputs are returned over a different channel.

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B. Model Description

The mathematical model to be analyzed in this paper is an idealized model of the ALOHA system with carrier sensing and blocking. This model consists of N users that are statistically independent and identical.

A block diagram of a single user is shown in Figure 1. At any time t, each user can be either in the active state or the blocked state. In the active state, a user tries to send a packet on the average every $\frac{1}{\alpha}$ seconds. Similarly, in the blocked state, a user tries to send a packet on the average every $\frac{1}{\beta}$ seconds. All packets are of the same length τ.

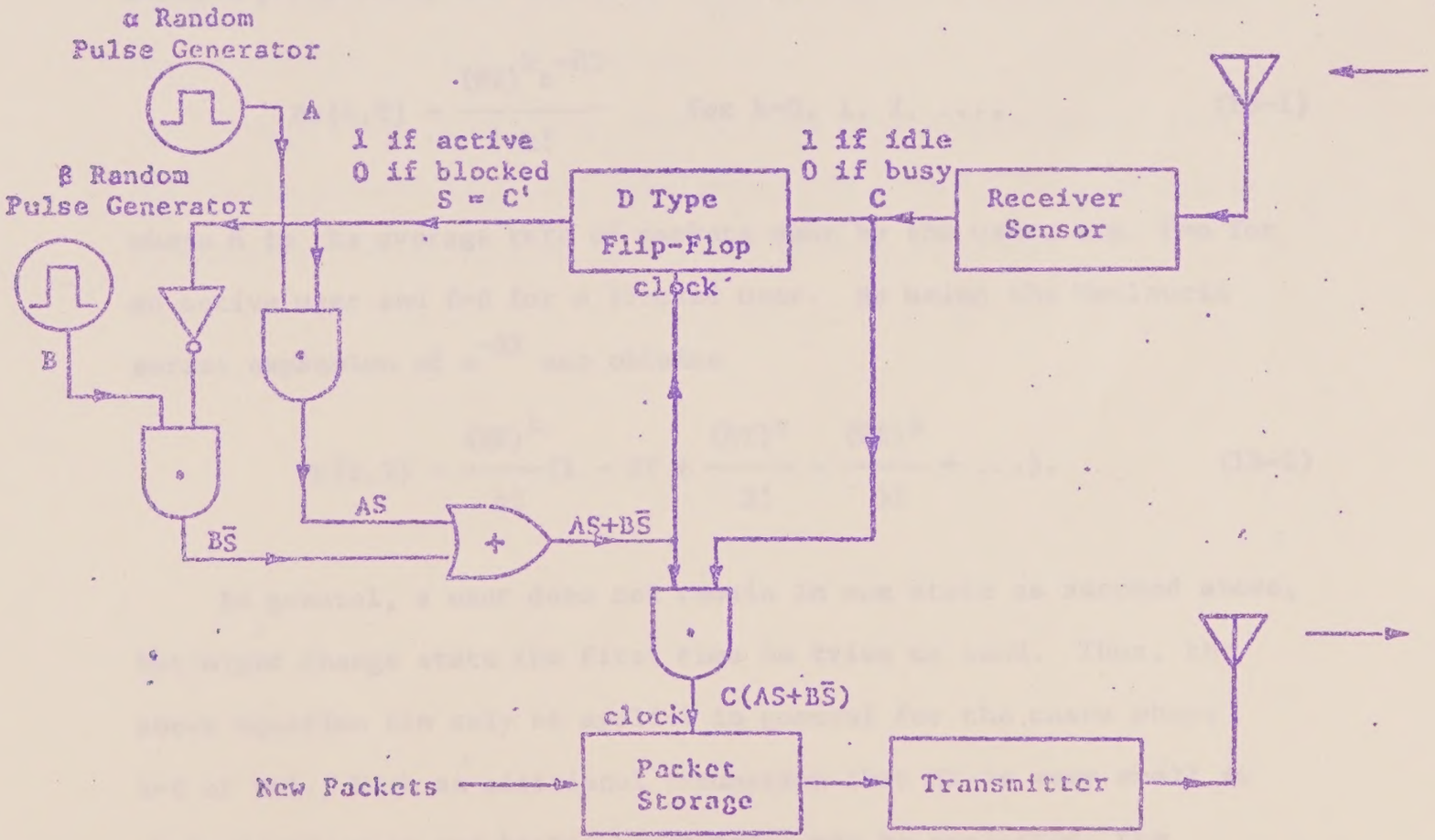
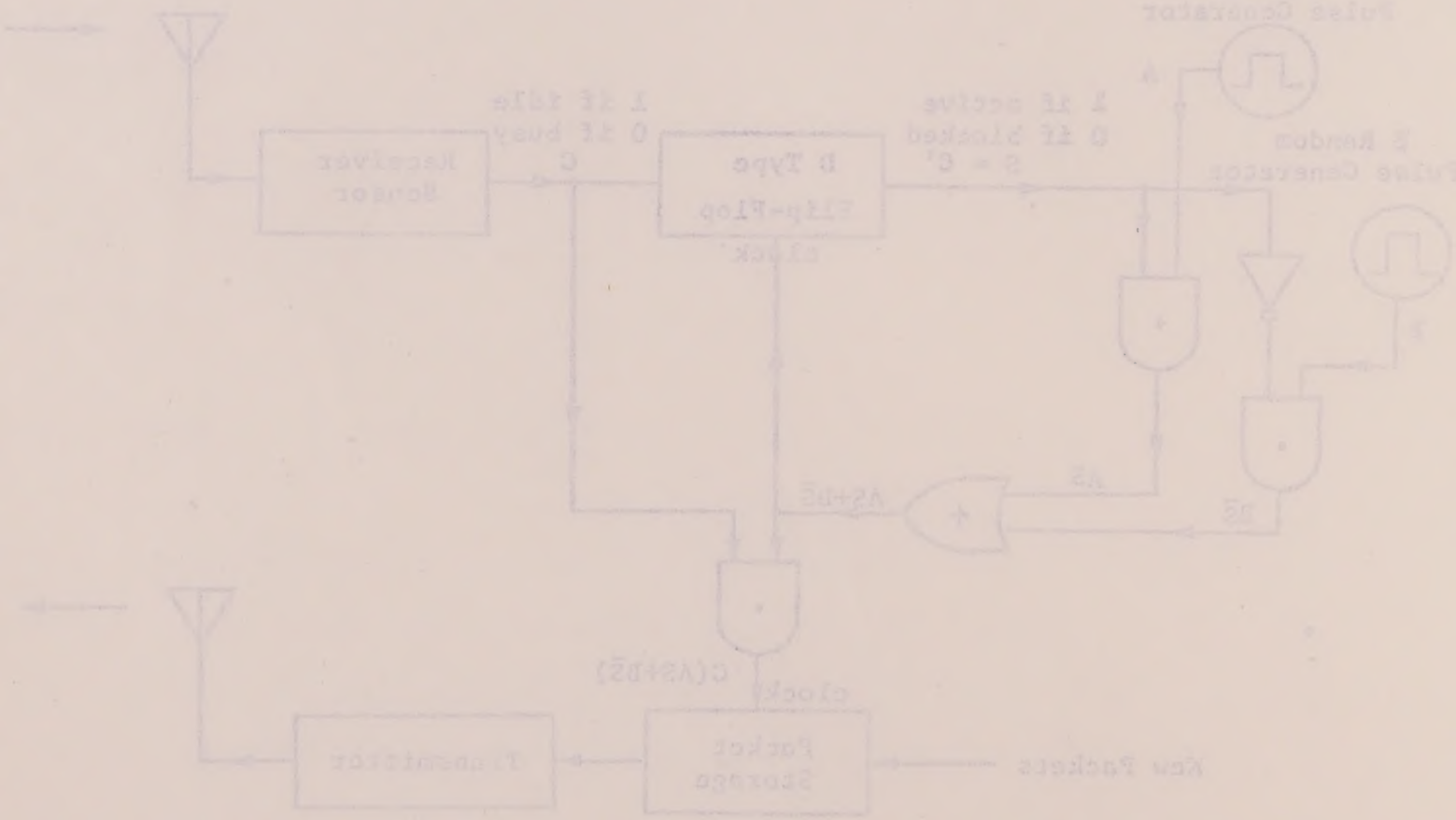


FIGURE 1

Block Diagram of a Single User


Receiver
Buffer
D Type
Flip-Flop
Clock
AS+BS
AS
BS
C(AS+BS)
clock
Packet
Storage
Transmitter
New Packets

Each user has carrier sensing, the ability to monitor the channel to know if it is busy. Any user who tries to send a packet when the channel is idle is successful and switches to the active state if not already active. Furthermore, any user who tries to send a packet when the channel is busy is unsuccessful and switches to the blocked state if not already blocked.

If a user remains in one state (i.e. if active, the channel is idle whenever he tries to send; or if blocked, the channel is busy whenever he tries to send), the probability that the user tries to send exactly k times in T seconds is assumed to be described by the Poisson probability distribution

$$\Pr[k,T] = \frac{(\bar{n}T)^k e^{-\bar{n}T}}{k!} \qquad \text{for } k=0, 1, 2, \ldots, \tag{1B-1}$$

where $\bar{n}$ is the average rate of packets sent by the user, e.g. $\bar{n}=\alpha$ for an active user and $\bar{n}=\beta$ for a blocked user. By using the Maclaurin series expansion of $e^{-\bar{n}T}$ one obtains

$$\Pr[k,T] = \frac{(\bar{n}T)^k}{k!}\left(1 - \bar{n}T + \frac{(\bar{n}T)^2}{2!} - \frac{(\bar{n}T)^3}{3!} + \ldots\right). \tag{1B-2}$$

In general, a user does not remain in one state as assumed above, but might change state the first time he tries to send. Thus, the above equation can only be applied in general for the cases where k=0 or k=1. With an additional assumption that $\bar{n}T$ is very small so that second order and higher order terms may be neglected, the probability that a user does not try in T seconds is

$$\Pr[0,T] \simeq 1 - \bar{n}T, \qquad (1B\text{-}3a)$$

the probability that a user tries exactly once in T seconds is

$$\Pr[1,T] \simeq \bar{n}T, \text{ and} \qquad (1B\text{-}3b)$$

the probability that a user tries more than once in T seconds is

$$\Pr[k,T] \simeq 0 \quad \text{for } k>1. \qquad (1B\text{-}3c)$$

Since each of the N users in the model are statistically independent the probability that any user tries is independent of the user's history and also independent of the other users. But since all the users are statistically identical they all have the same operating characteristics as described above.

Finally, the probability that any one of n active users tries to send in Δt seconds is $n \cdot a\Delta t$. Similarly, the probability that any one of m blocked users tries to send in Δt seconds is $m \cdot \beta\Delta t$.

II. ANALYSIS

A. Carrier Sensing with Blocking for a Single User System

At any time t, a single user system can assume only one of four states--an idle or busy channel with an active or blocked user. These states are denoted as:

$I_n(t) \triangleq$ idle channel with n blocked users at time t, and

$B_n(t) \triangleq$ busy channel with n blocked users at time t,

where n is either 0 or 1.

The random value of the state at time t is denoted as $X_N(t)$. The probability that $X_N(t)$ takes the value $x_n(t)$ is denoted

$$P[x_n(t)] \triangleq Pr[X_N(t)=x_n(t)],$$

where $x_n(t)$ is either $I_n(t)$ or $B_n(t)$ and where n is either 0 or 1. The probabilities of any combination of two states such as unions and intersections, or conditional probabilities are also denoted in a similar abbreviated notation. For example, the conditional probability of state $x_n(t+\Delta t)$ given the state $x'_n(t)$ is denoted as

$$P[x_n(t+\Delta t)|x'_n(t)] \triangleq Pr[X_N(t+\Delta t)=x_n(t+\Delta t)|X_N(t)=x'_n(t)].$$

The basic approach used to analyze this system is to express the probability of a particular state at time $t+\Delta t$ in terms of the probability of the state at time t. Then by choosing the time interval Δt arbitrarily small, so that second order and higher order terms can be neglected* effectively at most one event can occur during that time

*Higher order terms can be neglected because to obtain the differential equation one divides by Δt and takes the limit as $\Delta t \to 0$, thus all Δt^2 and higher order terms go to zero.

interval, and a differential equation for the probabilities of various states can be written. Since a packet may finish during the small interval if the channel was idle τ seconds earlier and a packet started during the time interval $(t-\tau, t-\tau+\Delta t)$, the state at time $t-\tau$ must be considered as well as the state at time t.

By using the law of total probability and the definition of conditional probability, the probability of the system being in a particular state x_n at time $t+\Delta t$ is

$$P[x_n(t+\Delta t)] = P[x_n(t+\Delta t)|I_0(t)]\cdot P[I_0(t)] + P[x_n(t+\Delta t)|I_1(t)]\cdot P[I_1(t)] + P[x_n(t+\Delta t)|B_0(t)]\cdot P[B_0(t)] + P[x_n(t+\Delta t)|B_1(t)]\cdot P[B_1(t)]. \quad (2A-1)$$

To facilitate the calculation of the probability of each state at time $t+\Delta t$ given the state at time t, Table 1 is constructed below which summarizes the events necessary to reach the resulting state at time $t+\Delta t$ from the given state at time t.

The blocks in each of the four columns correspond to each of the four terms in equation (2A-1), respectively.

Evaluation of the probabilities of the various events vary in degree of difficulty. The easiest are the blocks crossed out with an X since these require the occurance of more than one event during the arbitrarily small Δt, and the probability of multiple events in this interval is nearly zero. Next in order of difficulty are the blocks marked with a (1) which correspond to the probabilities that a user tries. The probability that a user tries is simply proportional to Δt, and is independent of whether the channel is idle or busy. The blocks marked (2) correspond

Given State at Time t

Resulting State at Time t+Δt	$I_0(t)$	$I_1(t)$	$B_0(t)$	$B_1(t)$
$I_0(t+\Delta t)$	active does not try (2)	blocked tries then packet finishes	packet finishes (3)	packet finishes then blocked tries then packet finishes
$I_1(t+\Delta t)$	active tries then active tries then packet finishes	blocked does not try (2)	active tries then packet finishes	packet finishes (3)
$B_0(t+\Delta t)$	active tries (1)	blocked tries (1)	packet does not finish and no one tries (4)	packet finishes then blocked tries
$B_1(t+\Delta t)$	active tries then active tries	blocked tries then active tries	active tries (1)	blocked tries, or no one tries and blocked does not finish (4)

TABLE 1

Events to Reach the Resulting State for One User

to the probabilities that a user does not start which is the complement of the probabilities that a user starts. The probability that a packet finishes and the probability that no event occurs which correspond to the blocks marked (3) and (4), respectively, are more difficult to evaluate. A discussion of the method used to evaluate these probabilities follows the next table which lists the conditional probabilities of the various events described in Table 1.

Resulting State at Time $t+\Delta t$ \ Given State at Time t	$I_0(t)$	$I_1(t)$	$B_0(t)$	$B_1(t)$
$I_0(t+\Delta t)$	$1-\alpha\Delta t$ (2)	0	$P[I_0(t+\Delta t)\|B_0(t)]$ (3)	0
$I_1(t+t)$	0	$1-\beta\Delta t$ (2)	0	$P[I_1(t+\Delta t)\|B_1(t)]$ (3)
$B_0(t+\Delta t)$	$\alpha\Delta t$ (1)	$\beta\Delta t$ (1)	$P[B_0(t+\Delta t)\|B_0(t)]$ (4)	0
$B_1(t+\Delta t)$	0	0	$\alpha\Delta t$ (1)	$P[B_1(t+\Delta t)\|B_1(t)]$ (4)

TABLE 2

Conditional Probabilities of the Resulting State for One User

For a packet to finish during the time interval $(t,t+\Delta t)$, the channel had to be idle at $t-\tau$, and a user (active or blocked) had to try within the next Δt seconds. Thus, the probability of the states at time $t-\tau$ must be considered.

The probability that a packet finishes during the time interval $(t,t+\Delta t)$ given that the channel was busy with no blocked user is denoted as $P[I_0(t+\Delta t)|B_0(t)]$. Instead of evaluating this conditionnal probability, consider the complete third term of equation (2A-1) with $x_n=I_0$ which is

$P[I_0(t+\Delta t)\&B_0(t)]$. By using the law of total probability and partitioning on the user starting in $(t-\tau, t-\tau+\Delta t)$, and then by conditioning on the earlier state, one obtains

$$P[I_0(t+\Delta t)\&B_0(t)] = \tag{2A-2}$$

$$\Pr[B_0(t)|I_0(t-\tau)\&{}^{\text{active tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}]\cdot\Pr[{}^{\text{active tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}|I_0(t-\tau)]\cdot P[I_0(t-\tau)]$$

$$+\Pr[B_0(t)|I_1(t-\tau)\&{}^{\text{blocked tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}]\cdot\Pr[{}^{\text{blocked tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}|I_1(t-\tau)]\cdot P[I_1(t-\tau)].$$

The first probability in each term is the probability that an active user does not try in the time interval $(t-\tau+\Delta t, t)$ which is clearly $e^{-\alpha(\tau-\Delta t)}$ from equation (1B-1). The second probability in each term is easy to evaluate since it is just the probability that a user tries. Thus, equation (2A-2) may be written as

$$P[I_0(t+\Delta t)\&B_0(t)] = \tag{2A-3}$$

$$e^{-\alpha(\tau-\Delta t)}\alpha\Delta t\, P[I_0(t-\tau)] + e^{-\alpha(\tau-\Delta t)}\beta\Delta t\, P[I_1(t-\tau)].$$

Now all the terms of the first row of Table 2 have been evaluated and equation (2A-1) for $x_n = I_0$ is

$$P[I_0(t+\Delta t)] = (1-\alpha\Delta t)P[I_0(t)] + \Delta t e^{-\alpha(\tau-\Delta t)}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}. \tag{2A-4}$$

By rearranging terms and dividing by Δt, one obtains,

$$\frac{P[I_0(t+\Delta t)]-P[I_0(t)]}{\Delta t} = \tag{2A-5}$$

$$-\alpha P[I_0(t)] + e^{-\alpha(\tau-\Delta t)}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}.$$

Next, by taking the limit as $\Delta t \to 0$, one finds the derivative

$$\dot{P}[I_0(t)] = \lim_{\Delta t \to 0} \frac{P[I_0(t+\Delta t)] - P[I_0(t)]}{\Delta t} = \tag{2A-6}$$

$$-\alpha P[I_0(t)] + e^{-\alpha\tau}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}.$$

The other term marked (3) in Table 1, is very similar to the previous one. The corresponding conditional probability in Table 2 is the probability that a packet finishes during the time interval $(t,t+\Delta t)$ given that the channel was busy with 1 blocked user and is denoted as $P[I_1(t+\Delta t)|B_1(t)]$. By similar arguments one obtains,

$$P[I_1(t+\Delta t)\&B_1(t)] = \tag{2A-7}$$

$$\Pr[B_1(t)|I_0(t-\tau)\&{}^{\text{active tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}]\cdot\Pr[{}^{\text{active tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}|I_0(t-\tau)]\cdot P[I_0(t-\tau)]$$

$$+ \Pr[B_1(t)|I_1(t-\tau)\&{}^{\text{blocked tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}]\cdot\Pr[{}^{\text{blocked tries}}_{\text{in}(t-\tau,t-\tau+\Delta t)}|I_1(t-\tau)]\cdot P[I_1(t-\tau)].$$

The first probability in each term is the probability that an active user tries at least once in the time interval $(t-\tau+\Delta t,t)$ which is $(1-e^{-\alpha(\tau-\Delta t)})$. The second probability in each term is the same as before and equation (2A-7) may be rewritten as

$$P[I_1(t+\Delta t)\&B_1(t)] = \tag{2A-8}$$

$$(1-e^{-\alpha(\tau-\Delta t)})\,\alpha\Delta t P[I_0(t-\tau)] + (1-e^{-\alpha(\tau-\Delta t)})\beta\Delta t P[I_1(t-\tau)].$$

All the terms of the second row of Table 2 have now been evaluated and can be substituted into equation (2A-1) with $x_n = I_1$. Then by rearranging terms and taking the limit as $\Delta t \to 0$, the derivative of $P[I_1(t)]$ is

$$\dot{P}[I_1(t)] = -\beta P[I_1(t)] + (1-e^{-\alpha\tau})\{\alpha P[I_0(t-\tau] + \beta P[I_1(t-\tau)]\}. \quad (2A-9)$$

Since the system can go into only one of four states from any given state, the sum of all the conditional probabilities in each column must be 1. Thus, the probability that no events occur during the time interval $(t,t+\Delta t)$ can be easily evaluated by considering its complement, which is just the sum of the other conditional probabilities in that column.

Hence, the conditional probability that no events occur given that the system was busy with no blocked users is

$$P[B_0(t+\Delta t)\,|\,B_0(t)] = \quad (2A-10)$$

$$1 - \frac{\Delta t e^{-\alpha(\tau-\Delta t)}}{P[B_0(t)]} \cdot \{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\} - \alpha\Delta t.$$

Now all the terms in the third row of Table 2 have been evaluated and by following earlier arguments, equation (2A-1) with $x_n = B_0$ can be solved to obtain

$$\dot{P}[B_0(t)] = \alpha P[I_0(t)] + \beta P[I_1(t)] - \alpha P[B_0(t)] \quad (2A-11)$$

$$- e^{-\alpha\tau}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}.$$

The other conditional probability marked (4) in Table 2 can also be evaluated by considering its complement to obtain

$$P[B_1(t+\Delta t)\,|\,B_1(t)] = \quad (2A-12)$$

$$1 - \frac{\Delta t}{P[B_1(t)]} \cdot (1-e^{-\alpha(\tau-\Delta t)}) \cdot \{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}.$$

Finally, by using the same earlier arguments, an equation for the derivative of $P[B_1(t)]$ obtained is:

$$\dot{P}[B_1(t)] = \alpha P[B_0(t)] - (1-e^{-\alpha\tau})\cdot\{\alpha P[I_0(t-\tau] + \beta P[I_1(t-\tau)]\}. \quad (2A\text{-}13)$$

This completes the evaluation of all the terms in Table 2. Summarizing, the differential equations obtained are:

$$\dot{P}[I_0(t)] = -\alpha P[I_0(t)] + e^{-\alpha\tau}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}, \quad (2A\text{-}6)$$

$$\dot{P}[I_1(t)] = -\beta P[I_1(t)] + (1-e^{-\alpha\tau})\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}, \quad (2A\text{-}9)$$

$$\dot{P}[B_0(t)] = \alpha P[I_0(t)] + \beta P[I_1(t)] - \alpha P[B_0(t)] - e^{-\alpha\tau}\{\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}, \text{ and} \quad (2A\text{-}11)$$

$$\dot{P}[B_1(t)] = \alpha P[B_0(t)] - (1-e^{-\alpha\tau})\,\alpha P[I_0(t-\tau)] + \beta P[I_1(t-\tau)]\}. \quad (2A\text{-}13)$$

B. Carrier Sensing with Blocking for an N user System

The analysis for an N user system is very similar to the single user system analysis. At any time t, an N user system can assume only one of 2(N+1) states--an idle or busy channel with n blocked users and N-n active users where n is an integer between zero and N. These states are denoted as before:

$I_n(t) \triangleq$ idle channel with n blocked users at t

$B_n(t) \triangleq$ busy channel with n blocked users at t,

where n is an integer and $0 \leq n \leq N$.

The probabilities of various states and their combinations are again expressed in abbreviated form as before. For example, the probability that the random value of the state at time t takes the value $x_n(t)$ is denoted as

$$P[x_n(t)] \triangleq \Pr[X_N(t)=x_n(t)].$$

The probability of a particular state at time t+Δt is expressed in terms of the probability of the state at time t. Then by choosing Δt arbitrarily small and by rearranging the equations, differential equations for the probabilities of various states are obtained.

By using the law of total probability and the definition of conditional probability and conditioning upon the 2(N+1) states at time t, one finds the probability of the system being in a particular state x_n at time t+Δt is

$$P[x_n(t+\Delta t)] = \sum_{\ell=0}^{N} \{P[x_n(t+\Delta t)|I_\ell(t)]P[I_\ell(t)] + P[x_n(t+\Delta t)|B_\ell(t)]P[B_\ell(t)]\}. \quad (2B-1)$$

Since Δt has been chosen arbitrarily small, effectively at most one event can occur during this time interval. Then, the above summation reduces to six terms, and

$$P[x_n(t+\Delta t)] = \sum_{\ell=n-1}^{n+1} \{P[x_n(t+\Delta t)|I_\ell(t)]P[I_\ell(t)] + P[x_n(t+\Delta t)|B_\ell(t)]P[B_\ell(t)]\}. \quad (2B-2)$$

To facilitate the calculation of the probability of each state at time $t+\Delta t$ given a particular state at time t, Table 3 is constructed. It summarizes the necessary events to go from the given state at time t to the resulting state at time $t+\Delta t$. The columns of the table represent the state of the system at time t and the rows of the table represent the state of the system at time $t+\Delta t$.

The blocks crossed with an X require the occurance of more than one event during the arbitrarily small interval Δt. Hence the probability of each of these blocks is effectively zero since the probability of multiple events in that interval is effectively zero. The blocks marked with a (1) are simply the prcoability that either a blocked or active user tries and this probability is simply proportional to Δt.

Since the number of blocked users can change by at most one, given a state with ℓ blocked users at time t, the resulting state at time $t+\Delta t$ can be only one of six possible states. Thus, the sum of the conditional probabilities of these six possible states at time $t+\Delta t$, given the state at time t must be 1. By using this fact, the block marked (2) can be easily evaluated as the complement of other probabilities in that column. The probability of the final blocks marked (3) and (4) are more difficult to evaluate since they include events where

Given State at Time t

Resulting State at Time t+Δt

	$I_{\ell-1}(t)$	$B_{\ell-1}(t)$	$I_\ell(t)$	$B_\ell(t)$	$I_{\ell+1}(t)$	$B_{\ell+1}(t)$
$I_{\ell-1}(t+\Delta t)$			blocked tries then packet finishes	packet finishes then blocked starts then packet finishes		
$B_{\ell-1}(t+\Delta t)$			blocked starts (1)	packet finishes then blocked starts		
$I_\ell(t+\Delta t)$	active tries then 2nd active tries then packet finishes	active tries then packet finishes	no one tries (2)	busy finishes (3)	blocked tries then packet finishes	packet finishes then blocked starts then packet finishes
$B_\ell(t+\Delta t)$	active tries then 2nd active tries (1)	active tries (1)	active tries (1)	no one tries and busy does not finish or blocked tries* (4)	blocked tries (1)	packet finishes then block starts
$I_{\ell+1}(t+\Delta t)$			active tries then 2nd active tries then packet finishes	active tries then packet finishes		
$B_{\ell+1}(t+\Delta t)$			active tries then 2nd active tries	active tries (1)		

*Applies only when $\ell=N$.

TABLE 3
Events to Reach the Resulting State for N User System

Given State at Time t

Resulting State at Time t+Δt	$I_{\ell-1}(t)$	$B_{\ell-1}(t)$	$I_\ell(t)$	$B_\ell(t)$	$I_{\ell+1}(t)$	$B_{\ell+1}(t)$
		$I_{\ell-1}(t+\Delta t)$	0	0		
		$B_{\ell-1}(t+\Delta t)$	$\ell\beta\Delta t$ (1)	0		
$I_\ell(t+\Delta t)$	0	0	$1-\ell\beta\Delta t$ $-(N-\ell)\alpha\Delta t$ (2)	$P[I_\ell(t+\Delta t)\mid B_\ell(t)]$ (3)	0	0
$B_\ell(t+\Delta t)$	0	$[N-(\ell-1)]\alpha\Delta t$ (1)	$(N-\ell)\alpha\Delta t$ (1)	$P[B_\ell(t+\Delta t)\mid B_\ell(t)]$ (4)	$(\ell+1)\beta\Delta t$ (1)	0
			0	0		
			0	$(N-\ell)\alpha\Delta t$ (1)		

TABLE 4

Conditioned Probabilities of the Resulting State for N Users

a packet finishes. A discussion of the method used to evaluate each of these blocks follows Table 4 which lists the conditional probabilities of the various events described in Table 3.

The block marked (3) involves the single event that a packet finishes during the time interval $(t,t+\Delta t)$. Again, to evaluate this event the state at $t-\tau$ must be considered; the channel must be idle at $t-\tau$ and a user (blocked or active) must start within the next Δt seconds for a packet to finish in $(t,t+\Delta t)$.

Instead of evaluating the conditional probability $P[I_\ell(t+\Delta t)|B_\ell(t)]$, consider the complete third term of equation (2B-2) with $x_n=I_\ell$ which is $P[I_\ell(t+\Delta t)\&B_\ell(t)]$. By using the law of total probability and partitioning on the number of blocked users at $t-\tau$ and the user starting in $(t-\tau,t-\tau+\Delta t)$, one finds

$$P[I_\ell(t+\Delta t)\&B_\ell(t)] = \sum_{k=0}^{N} [P(B_\ell(t)\&I_k(t-\tau)\&\text{ active starts in } (t-\tau,t-\tau+\Delta t)] + \sum_{k=0}^{N} P[B_\ell\&I_k(t-\tau)\&\text{ block starts in } (t-\tau,t-\tau+\Delta t)]. \tag{2B-3}$$

Then by conditioning on the state at $t-\tau$, one finds

$$P[I_\ell(t+\Delta t)\&B_\ell(t)] = \sum_{k=0}^{N} P[B_\ell(t)|B_k(t-\tau+\Delta t)]\cdot P[\text{active starts in}(t-\tau,t-\tau+\Delta t)|I_k(t-\tau)]\cdot P[I_k(t-\tau)] + \sum_{k=0}^{N} P[B_\ell(t)\ B_{k-1}(t-\tau+\Delta t)]\cdot P[\text{blocked starts in}(t-\tau,t-\tau+\Delta t)|I_k(t-\tau)]\cdot P[I_k(t-\tau)]. \tag{2B-4}$$

The second probability in each term is just the probability that a user starts given k blocked users at t-τ. The first probability of each term, however, is more difficult to evaluate. In general, it is the probability that m active users try in τ-Δt seconds given a busy channel throughout the τ-Δt seconds and with N-n active users at the start. Notice that as each active user tries he is blocked, thus the number of blocked users increases and changes the probability that an active user tries.

Since the channel is busy, each of the N-n active users must either try to transmit or not try to transmit. Now since the probability any user tries in τ seconds is just $e^{-\alpha\tau}$, the probability that he doesn't try is $1-e^{-\alpha\tau}$. In addition, since the N-n active users are statistically independent, one finds that $F_{m|n}(\tau)$, the probability that m active users try to send in τ seconds given that the channel is busy throughout the τ seconds and that there are n blocked users initially, is

$$F_{m|n}(\tau) = \begin{cases} \binom{N-n}{m} (1-e^{-\alpha\tau})^m (e^{-\alpha\tau})^{N-n-m}, & \tau \geq 0 \text{ and } 0 \leq m \leq N-n \\ 0, & \text{otherwise.} \end{cases} \qquad (2B-5)$$

Having evaluated this probability, the result can now be used in equation (2B-4) to obtain

$$P[I_\ell(t+\Delta t)\&B_\ell(t)] =$$

$$\sum_{k=0}^{N} [F_{\ell-k|k}(\tau-\Delta t)(N-k)\alpha\Delta t + F_{\ell-k+1|k-1}(\tau-\Delta t)k\beta\Delta t]\cdot P[I_k(t-\tau)]. \tag{2B-6}$$

Now all terms in the $I_\ell(t+\Delta t)$ row of Table 4 have been evaluated and equation (2B-2) with $x_n=I_\ell$ is

$$P[I_\ell(t+\Delta t)] = [1-\ell\beta\Delta t-(N-\ell)\alpha\Delta t]P[I_\ell(t)] \tag{2B-7}$$

$$+ \sum_{k=0}^{N} [F_{\ell-k|k}(\tau-\Delta t)(N-k)\alpha\Delta t + F_{\ell-k+1|k-1}(\tau-\Delta t)k\beta\Delta t]\cdot P[I_k(t-\tau)].$$

By rearranging terms and taking the limit as $\Delta t\to 0$, the equation for the derivative of $P[I_\ell(t)]$ obtained is

$$\dot{P}[I_\ell(t)] = [-\ell\beta-(N-\ell)\alpha]\cdot P[I_\ell(t)] \tag{2B-8}$$

$$+ \sum_{k=0}^{N} [\alpha(N-k)F_{\ell-k|k}(\tau) + k\beta F_{\ell-k+1|k-1}(\tau)]\cdot P[I_k(t-\tau)].$$

The probability of the events in the block marked (4), $P[B_\ell(t+\Delta t)|B_\ell(t)]$, is difficult to evaluate directly, but the sum of the conditional probabilities of the only six possible states at time $t+\Delta t$,

given the state at time t, must be 1. Thus, $P[B_\ell(t+\Delta t|B_\ell(t)]$ can be evaluated indirectly as the complement of the other probabilities in that column.

Hence, one finds

$$P[B_\ell(t+\Delta t)|B_\ell(t)] = 1 - (N-\ell)\alpha\Delta\tau \qquad (2B\text{-}9)$$

$$- \frac{1}{P[B_\ell(t)]} \sum_{k=0}^{N} [F_{\ell-k|k}(\tau-\Delta t)(N-k)\alpha\Delta t + F_{\ell-k+1|k-1}(\tau-\Delta t)k\beta\Delta t]\cdot P[I_k(t-\tau)].$$

Now all the terms in the $B_\ell(t+\Delta t)$ row of Table 4 have been evaluated and equation (2B-2) with $x_n = B_\ell$ is

$$P[B_\ell(t+\Delta t)] = [N-(\ell-1)]\alpha\Delta t P[B_{\ell-1}(t)] + (N-\ell)\alpha\Delta t P[I_\ell(t)] \qquad (2B\text{-}10)$$

$$+ [1-(N-\ell)\alpha\Delta t]P[B_\ell(t)] + (\ell+1)\beta\Delta t P[I_{\ell+1}(t)]$$

$$- \Delta t \sum_{k=0}^{N} [\alpha(N-k)F_{\ell-k|k}(\tau-\Delta t) + \beta k F_{\ell-k+1|k-1}(\tau-\Delta t)]P[I_k(t-\tau)].$$

By rearranging terms and taking the limit as $\Delta t \to 0$, one obtains the equation for the derivative of $P[B_\ell(t)]$

$$\dot{P}[B_\ell(t)] = [N-(\ell-1)]\alpha P[B_{\ell-1}(t)] + (N-\ell)\alpha P[I_\ell(t)] \qquad (2B\text{-}11)$$

$$- (N-\ell)\alpha P[B_\ell(t)] + (\ell+1)\beta P[I_{\ell+1}(t)]$$

$$- \sum_{k=0}^{N} [\alpha(N-k)F_{\ell-k|k}(\tau-\Delta t) + \beta k F_{\ell-k+1|k-1}(\tau)]\cdot P[I_k(t-\tau)].$$

This finally completes the evaluation of all the terms in Table 4. Summarizing, the differential operations obtained are:

$$\dot{P}[I_\ell(t)] = [-\ell\beta-(N-\ell)\alpha]\cdot P[I_\ell(t)] + \sum_{k=0}^{N} C_{\ell|k} P[I_k(t-\tau)],\ 0\le\ell\le N, \quad (2B-21)$$

and

$$\dot{P}[B_\ell(t)] = [N-(\ell-1)]\alpha P[B_{\ell-1}(t)] - (N-\ell)\alpha P[B_\ell(t)] + (N-\ell)\alpha P[I_\ell(t)]$$

$$+ (\ell+1)\beta P[I_{\ell+1}(t)] - \sum_{k=0}^{N} C_{\ell|k} P[I_k(t-\tau)],\ 0\le\ell\le N; \quad (2B-12)$$

where

$$C_{\ell|k} = (N-k)\alpha F_{\ell-k|k}(\tau) + k\beta F_{\ell-k+1|k-1}(\tau) \quad (2B-13)$$

and where

$$F_{m|n}(\tau) = \begin{cases} u(\tau)\binom{N-n}{m}(e^{\alpha\tau}-1)^m \cdot e^{-(N-n)\alpha\tau}, & \text{if } 0\le m\le N-n \\ 0, & \text{otherwise.} \end{cases} \quad (2B-14)$$

C. Solution of the Differential Equation

Let the partitioned 2(N+1) column vector of the probabilities of the possible system states at time t be denoted by

$$X(t) \triangleq \begin{bmatrix} P_I(t) \\ P_B(t) \end{bmatrix}, \text{ with derivative } \dot{X}(t) \triangleq \begin{bmatrix} \dot{P}_I(t) \\ \dot{P}_B(t) \end{bmatrix}$$

where $P_I(t)$ and $P_B(t)$ are the N+1 column vectors

$$\begin{bmatrix} P[I_0(t)] \\ P[I_1(t)] \\ \cdot \\ \cdot \\ \cdot \\ P[I_N(t)] \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} P[B_0(t)] \\ P[B_1(t)] \\ \cdot \\ \cdot \\ \cdot \\ P[B_N(t)] \end{bmatrix}, \text{ respectively.}$$

Then the parametric equations (2B-21) and (2B-24) can be written in vector form as

$$\dot{X}(t) = HX(t) + GX(t-\tau) \qquad (2C-1)$$

where the 2(N+1) x 2(N+1) matrices H and G can each be partitioned into four (N+1) x (N+1) matrices as

$$H = \begin{bmatrix} Q & \phi \\ R & S \end{bmatrix} \text{ and } G = \begin{bmatrix} C & \phi \\ -C & \phi \end{bmatrix}.$$

The matrix ϕ is the zero matrix, with the element in the ith row and jth column, $\phi_{i,j}$ equal to zero. The matrix Q is diagonal with elements

$$q_{i,j} = -[(i-1)\beta+(N-i+1)\alpha]\,\delta_{i-j}. \qquad (2C-2)$$

The matrices R and S are almost diagonal with elements:

$$r_{i,j} = (N-i+1)\alpha\delta_{i-j} + i\beta\delta_{i-j+1}, \text{ and} \tag{2C-3}$$

$$s_{i,j} = -(N-i+1)\alpha\delta_{i-j} + (N-j+1)\beta\delta_{i-j-1}, \tag{2C-4}$$

respectively. The matrix C is almost lower triangular with elements

$$c_{i,j} = (N-j+1)\alpha F_{i-j|j-1}(\tau) + (j-1)\beta F_{i-j+1|j-2}(\tau) \tag{2C-5}$$

where

$$F_{m|n}(\tau) = \begin{cases} u(\tau)\binom{N-n}{m}(1-e^{-\alpha\tau})^m e^{-(N-n-m)\alpha\tau}, & \text{if } 0 \leq n \leq N-n \\ 0, & \text{otherwise.} \end{cases} \tag{2B-15}$$

By performing the matrix multiplication by partitions, equation (2C-1) can be separated into the two equations:

$$\dot{P}_I(t) = QP_I(t) + CP_I(t-\tau), \tag{2C-6a}$$

$$\dot{P}_B(t) = RP_I(t) + SP_B(t) - CP_I(t-\tau). \tag{2C-6b}$$

Since equation (2C-6a) consists of only the probabilities that the channel is idle, it can be solved independently of equation (2C-6b). Then since both equations have the common term $CP_I(t-\tau)$, the latter can be solved easily by using the results of the former as

$$\dot{P}_B(t) = (R+Q)P_I(t) + SP_B(t) - \dot{P}_I(t). \tag{2C-7}$$

One procedure for solving the differential equations is to solve for X(t) in successive τ second time intervals. The starting condition

can be assumed to be $X(t) = X_0$, $-\tau<t\leq 0$. Then equation (2C-6a) and (2C-7) can be solved for $X(t)$ during the next time interval $0<t\leq\tau$ using X_0 as the initial condition. A similar procedure can be used to solve for $X(t)$ during subsequent τ second intervals using the solution during the immediately preceding τ seconds as initial conditions.

The complete solution by this method is the summation of all the piecewise solutions. This method is prohibitive when done by hand for even a small number of users, but it is feasible when programmed using conventional numerical methods.

A second procedure to find $X(t)$ uses frequency domain methods. By using Fourier transforms, the differential equation in time becomes a linear equation in frequency which can be solved for $\hat{X}(f)$, the Fourier transform of $X(t)$. Then $X(t)$ can be obtained by performing the inverse Fourier transform on $\hat{X}(f)$. This procedure would also be prohibitive by hand, but using Fast Fourier Transform subroutines, the computations are feasible.

For example, consider equation (2C-6a) with the initial condition $P_I(t) = P_{I_0}$ for $t<0$. Define $P_I^+(t) \triangleq P_I(t)U(t)$ and its derivative $\dot{P}_I^+(t) \triangleq P_I(t)\Delta(t) + \dot{P}_I(t)U(t)$ where $\Delta(t)$ and $U(t)$ are the $2(N+1)$ column vectors with each element equal to $\delta(t)$ or $u(t)$, respectively. By placing these terms into equation (2C-6a), one obtains

$$\dot{P}_I^+(t) - P_{I_0}\Delta(t) = QP_I^+(t) + C\{P_{I_0}\sqcap_\tau(t-\tfrac{1}{2}) + P_I^+(t-\tau)\}. \qquad (2C\text{-}8)$$

Next, by taking the Fourier transforms of the above equation one finds

$$j\omega\hat{P}_I^+(f) - P_{I_0} = Q\hat{P}_I^+(f) + CP_{I_0}e^{-j\omega\tau/2}\tau\,\mathrm{sinc}(f\tau) + e^{-j\omega\tau}C\hat{P}_I^+(f). \quad (2C\text{-}9)$$

where $\mathrm{sinc}\ x = \sin(\pi x)/\pi x$.

Then, by rearranging terms and solving for $P_I^+(f)$ one obtains

$$\hat{P}_I^+(f) = [j\omega I - Q - e^{-j\omega\tau}C]^{-1}\,[P_{I_0} + CP_{I_0}\,e^{-j\omega\tau/2}\tau\,\mathrm{sinc}(f\tau)]. \quad (2C\text{-}10)$$

Finally, the time domain solution $P_I^+(t)$ can be obtained by using the Fast Fourier Transform.

D. Steady State Solution

In steady state if one exists, $\dot{X}_\infty = \bar{0}$ and $X(t) = X(t-\tau) = X_\infty$. Thus equation (2C-1) reduces to the homogeneous system of linear equations,

$$(H+G)X_\infty = \bar{0} \qquad \text{(2D-1)}$$

where $\bar{0}$ is a vector of zeroes. Similarly, the partitioned equations (2C-6a) and (2C-7) reduce to

$$(Q+C)P_I = \bar{0}, \text{ and} \qquad \text{(2D-2a)}$$

$$(R+Q)P_I + SP_B = \bar{0}. \qquad \text{(2D-2b)}$$

A system of linear equations in n unknowns has a unique solution if there are n independent equations relating the unknowns. However, upon closer examination of vector equations (2D-2a) and (2D-2b), one finds that there is at least one redundant equation in each.

First consider equation (2D-2a) which is the vector form of N+1 equations in N+1 unknowns. Define a scalar γ_I to be the sum of these N+1 equations. Then,

$$\gamma_I = \sum_{\ell=0}^{N} \dot{P}(I_\ell) = \sum_{\ell=0}^{N} [-\ell\beta - (N-\ell)\alpha]\cdot P(I_\ell) + \sum_{\ell=0}^{N}\sum_{k=0}^{N} C_{\ell|k} P(I_k). \qquad \text{(2D-3)}$$

By interchanging the order of summation, the two summations can be combined and $P(I_\ell)$ can be factored from the sum to obtain

$$\gamma_I = \sum_{\ell=0}^{N} [-\ell\beta - (N-\ell)\alpha + \sum_{k=0}^{N} C_{k|\ell}]\cdot P(I_\ell). \qquad \text{(2D-4)}$$

Next, by substituting for $C_{k|\ell}$ one finds

$$\gamma_I = \sum_{\ell=0}^{N} \left[-\ell\beta - (N-\ell)\alpha + (N-\ell)\alpha \sum_{k=0}^{N} F_{k-\ell|\ell} + \ell\beta \sum_{k=0}^{N} F_{k-\ell+1|\ell-1}\right] \cdot P(I_\ell). \tag{2D-5}$$

The two inner summations are the sum of all the probabilities of the possible number of active users that may try in τ seconds with a blocked channel with a given number of blocked users at the start. Thus, these summations are equal to unity and equation (2D-5) reduces to

$$\gamma_I = \sum_{\ell=0}^{N} 0 \cdot P(I_\ell). \tag{2D-6}$$

Since the coefficients of $P(I_\ell)$ are zero for $\ell=1,2,\ldots,N$, one equation is redundant and it can be written as the negative of the sum of the other equations.

Next consider equation (2D-2b) and similarly define γ_B to be the sum of these N+1 homogeneous equations. Then, by similar procedures

$$\gamma_B = \sum_{\ell=0}^{N} [N-(\ell-1)]\alpha P[B_{\ell-1}] - \sum_{\ell=0}^{N} (N-\ell)\alpha P[B_\ell] \tag{2D-7}$$

$$+ \sum_{\ell=0}^{N} (\ell+1)\beta P[I_{\ell+1}] - \sum_{\ell=0}^{N} \ell\beta P[I_\ell].$$

By performing a change of variables, it becomes clear that the first two terms and the last two terms are pairwise identical and cancel each other. Thus, equation (2D-7) can be written as

$$\gamma_B = \sum_{\ell=0}^{N} \{0 \cdot P[I_\ell] + 0 \cdot P[B_\ell]\}. \qquad (2D\text{-}8)$$

Another equation is redundant.

Therefore, to obtain a unique steady state solution for this system, at least two additional equations are needed. One additional equation is a normalization condition that the sum of the probabilities of ℓ blocked users for all integers ℓ $(0 \leq \ell \leq N)$ is unity, which can be mathematically stated

$$\sum_{\ell=0}^{N} \{P[I_\ell] + P[B_\ell]\} = 1. \qquad (2D\text{-}9)$$

Another equation that is valid for steady state equates the rate of packets entering the channel to the rate of packets leaving the channel. Consider the system in steady state for a long time interval $T \gg \tau$. The channel is idle with ℓ blocked users for $T \cdot P(I_\ell)$ seconds during the interval T. The average number of tries per second by both active and blocked users given ℓ blocked users is $[(N-\ell)\alpha+\ell\beta]$. If the channel is idle with ℓ blocked users when an active or blocked user tries, he is successful and the channel becomes busy. Thus the average number of successful tries in the T second interval is

$$\sum_{\ell=0}^{N} [(N-\ell)\alpha + \ell\beta]TP(I_\ell).$$

Next consider the average number of packets occupying the channel during a long time interval $T \gg \tau$. The channel is busy for an average

of $T \cdot \sum_{\ell=0}^{N} P(B_\ell)$ seconds during the interval T. Since each packet is τ seconds long, the average number of packets occupying the channel during the T second interval is

$$\frac{1}{\tau} T \cdot \sum_{\ell=0}^{N} P[B_\ell].$$

Equating this term to the average number of packets starting in the T second interval and dividing both sides of the equation by $\frac{T}{\tau}$ one obtains the rate equation,

$$\sum_{\ell=0}^{N} \tau[(N-\ell)\alpha + \ell\beta]P(I_\ell) = \sum_{\ell=0}^{N} P(B_\ell). \qquad (2D\text{-}10)$$

By rearranging terms and using equation (2D-9) in the form $\sum_{\ell=0}^{N} P(B_\ell) = 1 - \sum_{\ell=0}^{N} P(I_\ell)$, the above equation can be written as

$$\sum_{\ell=0}^{N} \{1 + [(N-\ell)\alpha + \ell\beta]\tau\}P(I_\ell) = 1. \qquad (2D\text{-}11)$$

Since equation (2D-11) is a linear combination of only the probabilities of an idle channel with ℓ blocked users with $\ell = 0,1,\ldots,N$, this equation can be combined with the first N equations (2D-2a) to hopefully find a unique solution for the probabilities of an idle channel with ℓ blocked users.

Define $(Q + C)'$ to be the $(N+1) \times (N+1)$ matrix $(Q + C)$ of equation (2D-2a) with the last row replaced by the coefficients of equation (2D-11).

Then the new matrix equation obtained is

$$(Q + C)'P_I = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (2D\text{-}12)$$

The matrix $(Q + C)'$ is almost lower triangular and if the (N+1) equations are independent, then the inverse exists and

$$P_I = [(Q + C)']^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (2D\text{-}13)$$

By using the normalization condition equation and the solution P_I with the first N equations of vector equation (2D-2b), one hopefully obtains a unique solution for P_B. Define S' and $(R - C)'$ to be the (N+1)x(N+1)matrices S and $(R - C)$ of equation (2D-2a) with the last row replaced by the coefficients of equation (2D-12). Then a new matrix equation obtained is

$$(R - C)'P_I + S'P_B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} . \quad (2D\text{-}14)$$

Matrix S' is lower triangular and if there are N+1 independent equations, matrix S' is invertible and the unique solution is

$$P_B = [S']^{-1}\left[\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} - (R - C)'P_I \right]. \quad (2D\text{-}15)$$

E. Efficiency Parameters

There are several measures which are useful to compare the efficiency of this system with other ALOHA-like systems. These measures are: 1) the average rate of throughput, 2) the average delay before a successful transmission, and 3) the average number of blocked users.

Consider the average rate of throughput $\bar{a}$ which is defined as the expected number of packets that occupy the channel per second in steady state. This quantity has been calculated previously in equation (2D-10) to be

$$\bar{a} = \frac{1}{\tau} \sum_{\ell=0}^{N} P(B_\ell) = \frac{1}{\tau} P(B). \qquad (2E\text{-}1)$$

Next consider the average delay before a successful transmission, $\bar{d}$, which will be defined as the average waiting time of an additional test user with the same statistics as the other users. The other users, however, are independent of this test user. If the channel is idle when the test user tries to transmit, he is successful and there is no delay and the probability of no delay is $[1-P(B)]$. However, if the channel is busy when he tries to transmit, he will switch to the blocked state if he was not already blocked. In the blocked state, the test user tries at an average rate of β times per second or equivalently after an average delay of $\frac{1}{\beta}$ seconds. If the channel is now idle, he is successful, and the transmission delay is $\frac{1}{\beta}$ seconds and the probability of this delay is $[1-P(B)]P(B)$. By similar arguments, the probability that the test user is successful on the $(k+1)^{st}$ try is $P(B)^k\,[1-P(B)]$. Thus, the average delay before a successful transmission is

$$\bar{d} = \sum_{k=1}^{\infty} \frac{k}{\beta}[1-P(B)]P(B)^k = \frac{P(B)}{\beta[1-P(B)]} = \frac{\bar{a}\tau}{\beta[1-\bar{a}\tau]} . \qquad (2E-2)$$

Finally, consider the average number of blocked users, $\bar{n}$, in steady state which can be written down immediately as

$$\bar{n} = \sum_{\ell=0}^{N} \ell[P(I_\ell)+P(B_\ell)] . \qquad (2E-3)$$

III. RESULTS AND DISCUSSION

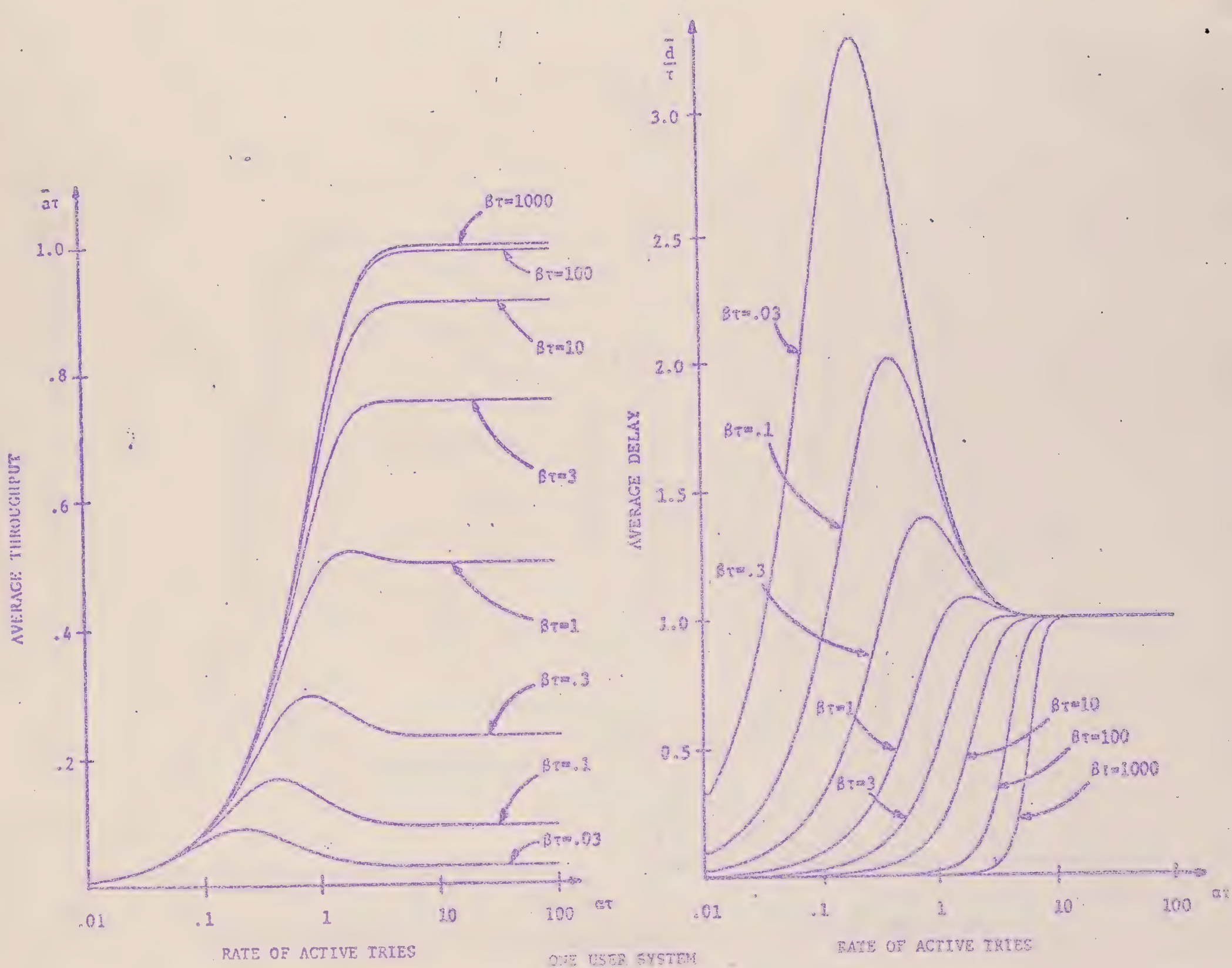
A. One and Two User ALOHA System

In the simple cases for one and two users, steady state vector equations (2D-12) and (2D-14) have unique solutions. The efficiency parameters are easy to calculate once the steady state solution is obtained.

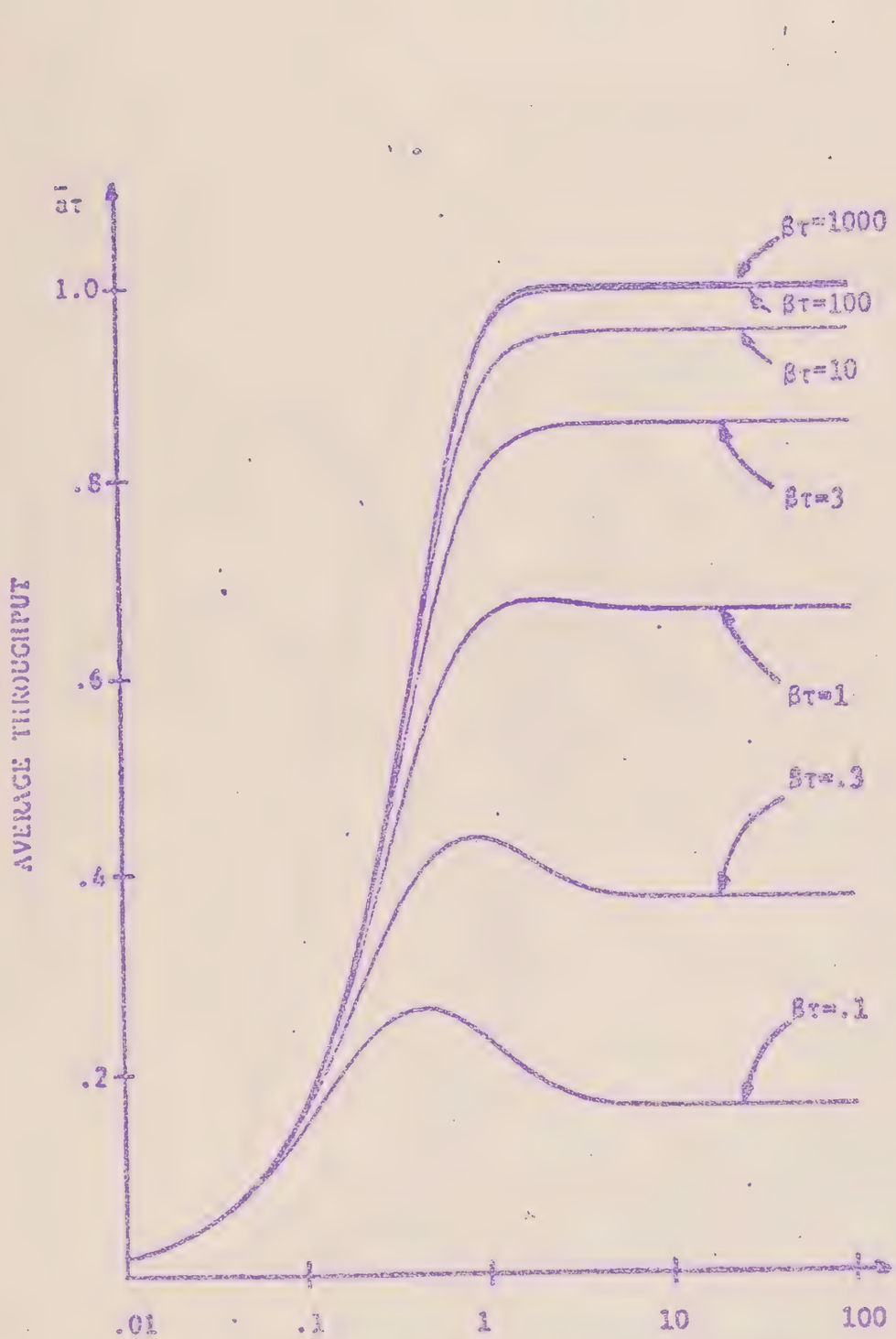
Normalized curves of the throughput, $\bar{a}$, the average delay of an additional test user, $\bar{d}$, and the average number of blocked users, $\bar{n}$, are plotted on the following pages. The calculations and graphs were made by using a Hewlett Packard 9100B Programmable Calculator and 9125B Calculator Plotter.

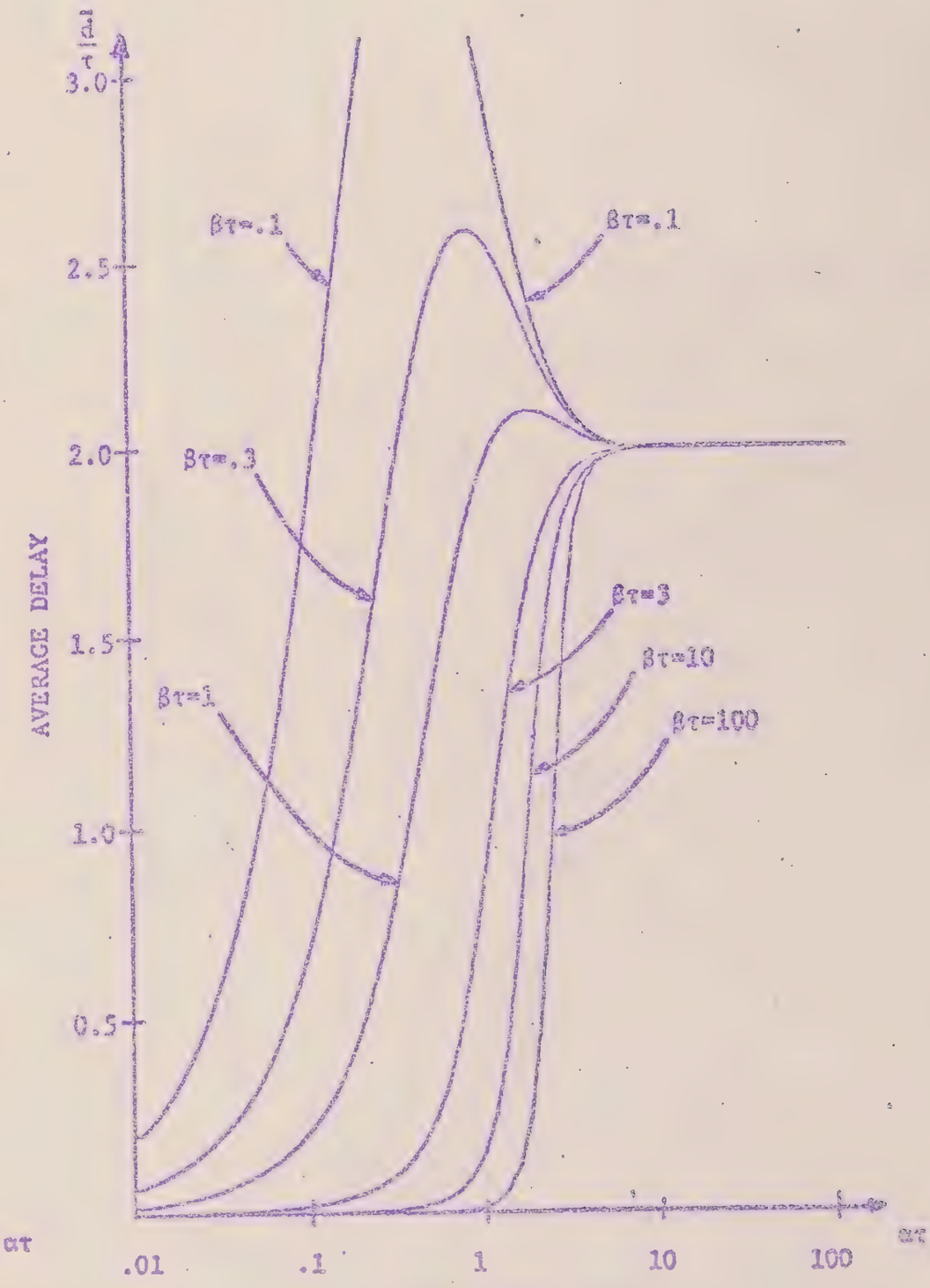
For low values of $\beta\tau$, the curves for $\bar{a}\tau$ and $\bar{d}/\tau$ reached local maximums at corresponding values of $\alpha\tau$ before settling to a constant value. A physical reason for this behavior is not known at this time. For higher values of β, the peaks became less prominent. From the information in these curves, one should select $\beta\tau$ large to maximize throughput and minimize delay. The maximum throughput for very large $\beta\tau$ with $\alpha\tau$ sufficiently large, approaches $\frac{1}{\tau}$. For a given value of $\beta\tau$, both throughput and delay increase as $\alpha\tau$ increases up to their respective peaks and then drop to a constant value. Thus, one must accept additional delay to obtain increased throughput.

The curves of $\bar{n}/N$ are S shaped curves which approach unity for large values of $\alpha\tau$. In this undesirable condition, all the users are usually in the blocked state. The curve also seems to show that for a given value of $\alpha\tau$, there is a lower bound on the value of $\frac{\bar{n}}{N}$, which is approached as $\beta\tau$ becomes large.

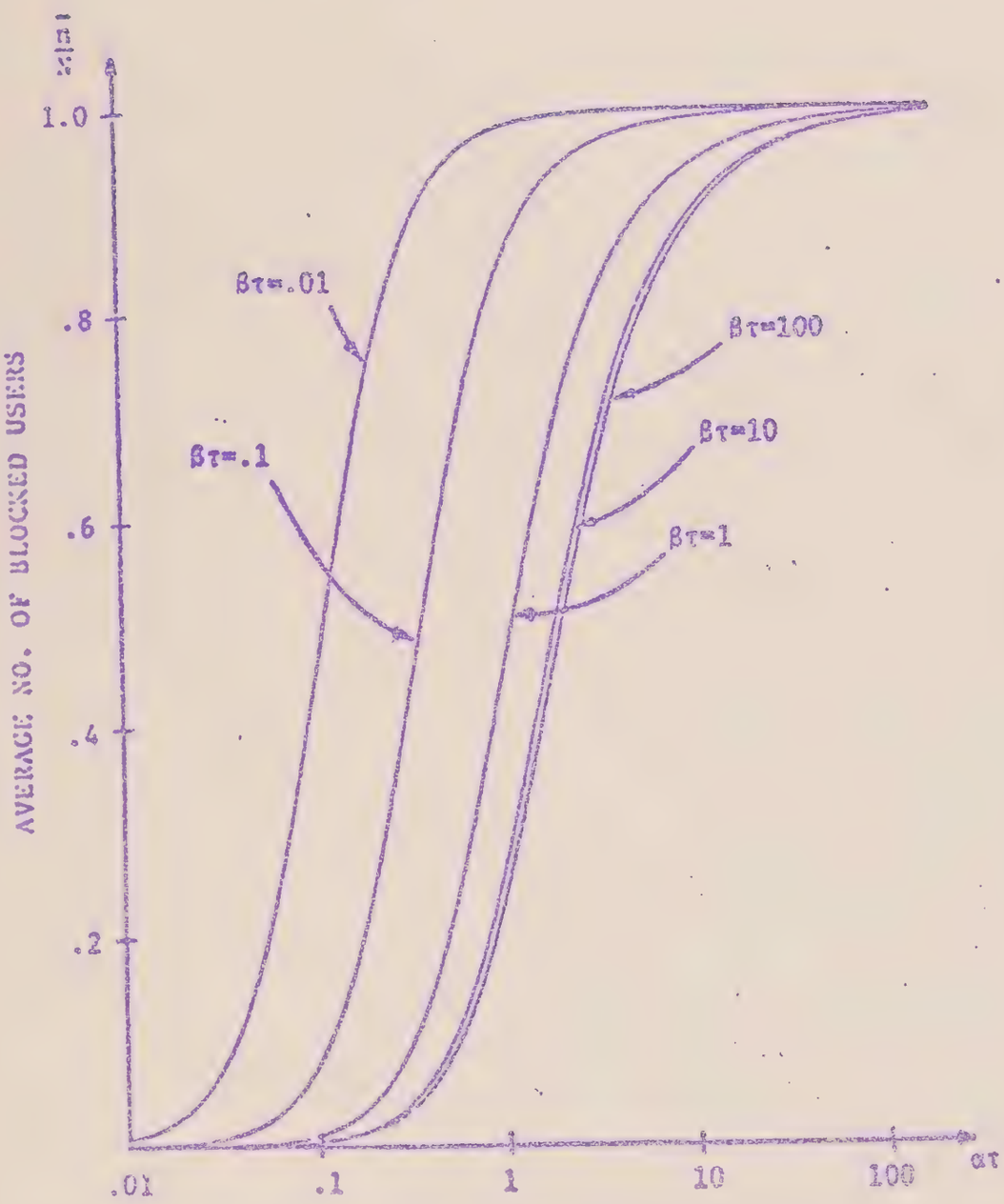


ONE USER SYSTEM

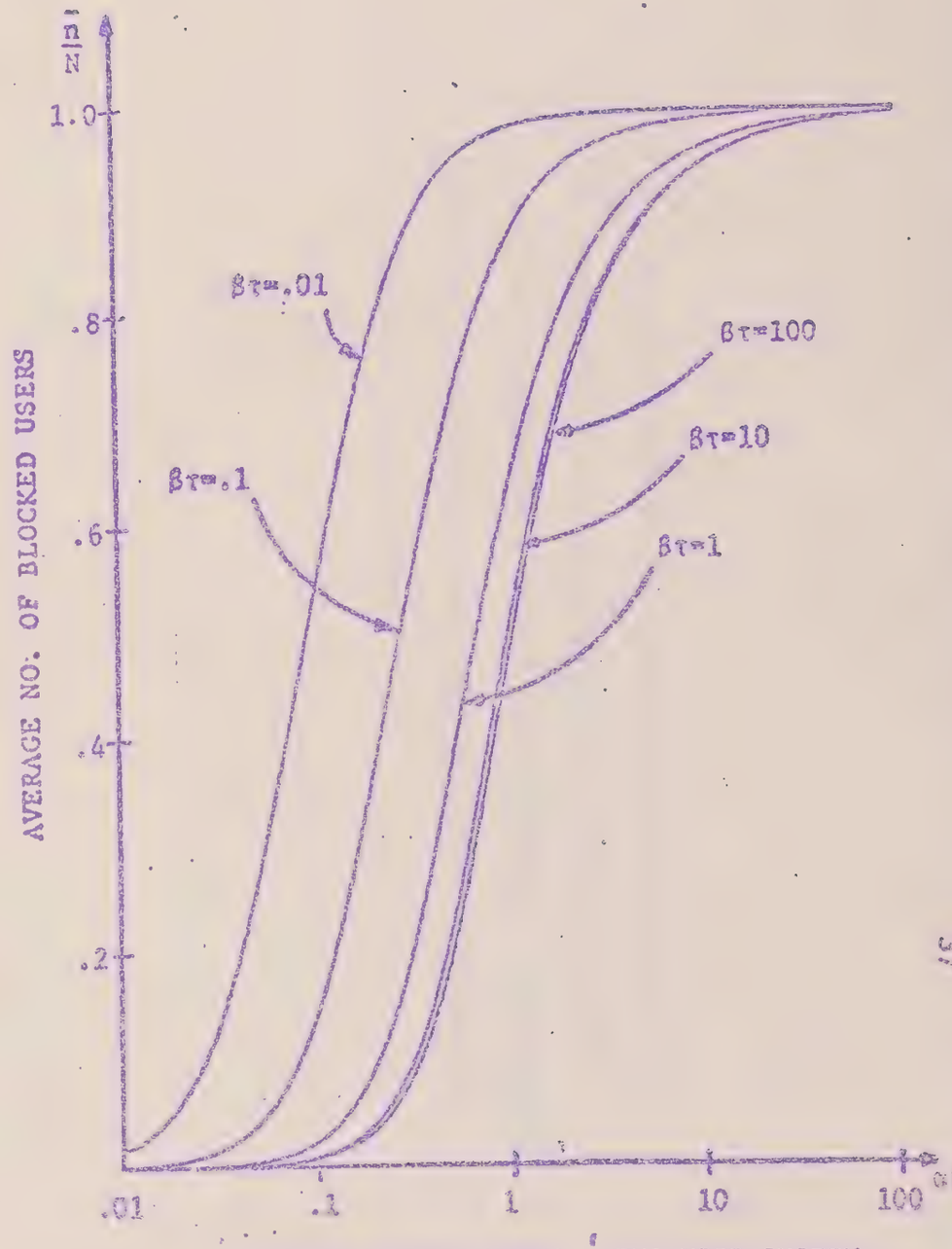




TWO USER SYSTEM



RATE OF ACTIVE TRIES - ONE USER SYSTEM



RATE OF ACTIVE TRIES - TWO USER SYSTEM

In all the curves, δ_T should be selected as large as possible to improve the system performance in that parameter. However, in the selection of α_T, one must compromise among the parameters. To decrease delay and the number of blocked users, one must decrease α_T which in turn decreases throughput and vice versa.

B. N User ALOHA System

In the general N user case, a system of 2(N+1) first order differential equations were obtained which describe the dynamic behavior of the model. The equations are solvable by conventional numerical methods or frequency domain methods.

If a steady state does exist, then the solution obtained by solving vector equations (2D-12) and (2D-14) is the steady state solution. These equations were uniquely solvable for the one and two user cases and it is probably possible to obtain a unique solution for systems with more than one user although this has not been proven. Efficiency parameter curves for such a general N user system are probably similar to the curves for the simple cases examined in this paper. Therefore, the criterion for selecting α_T and β_T for optimum performance would be the same.

It is possible that a steady state solution does not exist for all values of α_T and β_T as implied by the curves of the simple examples. In fact, a steady state might not exist at peaks and constant values in the curves of $\bar{a}$ and $\bar{d}$. The existence, however, can be checked by either solving the differential equations or by doing a computer simulation of the system in operation.

C. Conclusions

The mathematical model analyzed in this paper is a system with ideal carrier sensing and with statistically independent and identical users. This model cannot be physically realized exactly since there are various approximations which must be made in real systems. For example, radio transmission propagations delays and finite switching speeds of real devices may cause the packets of two users to collide if both sensed an idle channel almost simultaneously. Furthermore, packets from keypunch terminals come at various rates depending on the person using it.

Nevertheless, this mathematical model is an approximation of a real system and if the model is good it will behave almost like the real system.

The analysis shows that the proposed scheme with blocking and carrier sense is a feasible way to implement the time sharing channels of an ALOHA system.

The general procedure used to analyze this system is also significant. By expressing the probability of each system state at time $t+\Delta t$ in terms of the probabilities of the system states at time t, a set of differential equations can be obtained. The equations describe the dynamic behavior of the system and can be solved by numerical methods to obtain both transient and steady state solutions. Finally, the various efficiency parameters which are important in the selection of the values of the various system parameters can be obtained.

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